CHAPTER 5 CONCRETE FORMULAS

CHAPTER FIVE

REINFORCED CONCRETE

When working with reinforced concrete and when designing reinforced concrete structures, the American Concrete Institute (ACI) Building Code Requirements for Reinforced Concrete, latest edition, is widely used. Future references to this document are denoted as the ACI Code. Likewise, publications of the Portland Cement Association (PCA) find extensive use in design and construction of reinforced concrete structures.

Formulas in this chapter cover the general principles of reinforced concrete and its use in various structural applications. Where code requirements have to be met, the reader must refer to the current edition of the ACI *Code* previously mentioned. Likewise, the PCA publications should also be referred to for the latest requirements and recommendations.

WATER/CEMENTITIOUS MATERIALS RATIO

The water/cementitious (w/c) ratio is used in both tensile and compressive strength analyses of Portland concrete cement. This ratio is found from

$$\frac{w}{c} = \frac{w_m}{w_c}$$

where w_m = weight of mixing water in batch, lb (kg); and w_c = weight of cementitious materials in batch, lb (kg).

The ACI *Code* lists the typical relationship between the w/c ratio by weight and the compressive strength of concrete. Ratios for non-air-entrained concrete vary between 0.41 for

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a 28-day compressive strength of 6000 lb/in² (41 MPa) and 0.82 for 2000 lb/in² (14 MPa). Air-entrained concrete w/c ratios vary from 0.40 to 0.74 for 5000 lb/in² (34 MPa) and 2000 lb/in² (14 MPa) compressive strength, respectively. Be certain to refer to the ACI *Code* for the appropriate w/c value when preparing designs or concrete analyses.

Further, the ACI *Code* also lists maximum w/c ratios when strength data are not available. Absolute w/c ratios by weight vary from 0.67 to 0.38 for non-air-entrained concrete and from 0.54 to 0.35 for air-entrained concrete. These values are for a specified 28-day compressive strength f'_c in lb/in² or MPa, of 2500 lb/in² (17 MPa) to 5000 lb/in² (34 MPa). Again, refer to the ACI *Code* before making any design or construction decisions.

Maximum w/c ratios for a variety of construction conditions are also listed in the ACI *Code*. Construction conditions include concrete protected from exposure to freezing and thawing; concrete intended to be watertight; and concrete exposed to deicing salts, brackish water, seawater, etc. Application formulas for w/c ratios are given later in this chapter.

JOB MIX CONCRETE VOLUME

A trial batch of concrete can be tested to determine how much concrete is to be delivered by the job mix. To determine the volume obtained for the job, add the *absolute* volume V_a of the four components—cements, gravel, sand, and water.

Find the V_a for each component from

$$V_a = \frac{W_L}{(\text{SG})W_u}$$

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- where V_a = absolute volume, ft³ (m³)
 - W_L = weight of material, lb (kg)
 - SG = specific gravity of the material
 - w_u = density of water at atmospheric conditions (62.4 lb/ft³; 1000 kg/m³)

Then, job yield equals the sum of V_a for cement, gravel, sand, and water.

MODULUS OF ELASTICITY OF CONCRETE

The modulus of elasticity of concrete E_c —adopted in modified form by the ACI *Code*—is given by

 $E_c = 33w_c^{1.5}\sqrt{f_c'} \quad \text{lb/in}^2 \text{ in USCS units}$ $= 0.043w_c^{1.5}\sqrt{f_c'} \quad \text{MPa in SI units}$

With normal-weight, normal-density concrete these two relations can be simplified to

$$\begin{split} E_c &= 57,000 \sqrt{f_c'} \quad \text{lb/in}^2 \text{ in USCS units} \\ &= 4700 \sqrt{f_c'} \quad \text{MPa in SI units} \end{split}$$

where $E_c =$ modulus of elasticity of concrete, lb/in² (MPa); and $f'_c =$ specified 28-day compressive strength of concrete, lb/in² (MPa).

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TENSILE STRENGTH OF CONCRETE

The tensile strength of concrete is used in combined-stress design. In normal-weight, normal-density concrete the tensile strength can be found from

> $f_r = 7.5 \sqrt{f_c'}$ lb/in² in USCS units $f_r = 0.7 \sqrt{f_c'}$ MPa in SI units

REINFORCING STEEL

American Society for Testing and Materials (ASTM) specifications cover renforcing steel. The most important properties of reinforcing steel are

- **1.** Modulus of elasticity E_s , lb/in² (MPa)
- 2. Tensile strength, lb/in² (MPa)
- **3.** Yield point stress f_v , lb/in² (MPa)
- 4. Steel grade designation (yield strength)
- 5. Size or diameter of the bar or wire

CONTINUOUS BEAMS AND ONE-WAY SLABS

The ACI *Code* gives approximate formulas for finding shear and bending moments in continuous beams and oneway slabs. A summary list of these formulas follows. They are equally applicable to USCS and SI units. Refer to the ACI *Code* for specific applications of these formulas.

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For Positive Moment

End spans	
If discontinuous end is unrestrained	$w l_n^2 / 11$
If discontinuous end is integral with the support	$w l_n^2 / 14$
Interior spans	$w l_n^2 / 16$

For Negative Moment

Negative moment at exterior face of first interior support	
Two spans	$w l_n^2 / 9$
More than two spans	$w l_n^2 / 10$
Negative moment at other faces of interior supports	$w l_n^2 / 11$
Negative moment at face of all supports for (<i>a</i>) slabs with spans not exceeding 10 ft (3 m) and (<i>b</i>) beams and girders where the ratio of sum of column stiffness to beam stiffness exceeds 8 at each end of the span	$w l_n^2 / 12$
Negative moment at interior faces of exterior supports, for members built integrally with their supports	
Where the support is a spandrel beam or girder	$w l_n^2 / 24$
Where the support is a column	$w l_n^2 / 16$

Shear Forces

Shear in end members at first interior support	$1.15 \ w l_n/2$
Shear at all other supports	$wl_n/2$

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End Reactions

Reactions to a supporting beam, column, or wall are obtained as the sum of shear forces acting on both sides of the support.

DESIGN METHODS FOR BEAMS, COLUMNS, AND OTHER MEMBERS

A number of different design methods have been used for reinforced concrete construction. The three most common are *working-stress design, ultimate-strength design,* and *strength design method.* Each method has its backers and supporters. For actual designs the latest edition of the ACI *Code* should be consulted.

Beams

Concrete beams may be considered to be of three principal types: (1) rectangular beams with tensile reinforcing only, (2) T beams with tensile reinforcing only, and (3) beams with tensile and compressive reinforcing.

Rectangular Beams with Tensile Reinforcing Only. This type of beam includes slabs, for which the beam width b equals 12 in (305 mm) when the moment and shear are expressed per foot (m) of width. The stresses in the concrete and steel are, using working-stress design formulas,

$$f_c = \frac{2M}{kjbd^2}$$
 $f_s = \frac{M}{A_sjd} = \frac{M}{pjbd^2}$

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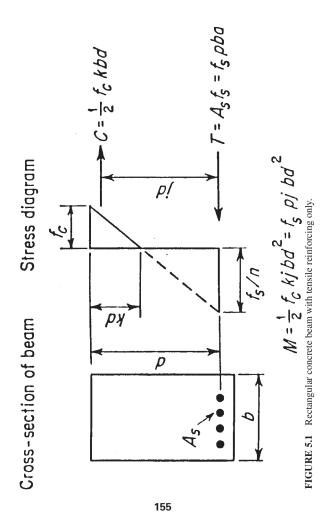
- where b = width of beam [equals 12 in (304.8 mm) for slab], in (mm)
 - *d* = effective depth of beam, measured from compressive face of beam to centroid of tensile reinforcing (Fig. 5.1), in (mm)
 - M = bending moment, lb · in (k · Nm)
 - $f_c = \text{compressive stress in extreme fiber of concrete,}$ lb/in² (MPa)
 - $f_s =$ stress in reinforcement, lb/in² (MPa)
 - $A_s = \text{cross-sectional}$ area of tensile reinforcing, in² (mm²)
 - j = ratio of distance between centroid of compression and centroid of tension to depth d
 - k = ratio of depth of compression area to depth d
 - p = ratio of cross-sectional area of tensile reinforcing to area of the beam (= A_s/bd)

For approximate design purposes, *j* may be assumed to be $\frac{7}{8}$ and *k*, $\frac{1}{3}$. For average structures, the guides in Table 5.1 to the depth *d* of a reinforced concrete beam may be used.

For a balanced design, one in which both the concrete and the steel are stressed to the maximum allowable stress, the following formulas may be used:

$$bd^2 = \frac{M}{K}$$
 $K = \frac{1}{2}f_ekj = pf_sj$

Values of K, k, j, and p for commonly used stresses are given in Table 5.2.



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Member	d
Roof and floor slabs	1/25
Light beams	1/15
Heavy beams and girders	<i>l</i> /12– <i>l</i> /10

TABLE 5.1 Guides to Depth d ofReinforced Concrete Beam[†]

 $^{\dagger}l$ is the span of the beam or slab in inches (millimeters). The width of a beam should be at least l/32.

T Beams with Tensile Reinforcing Only. When a concrete slab is constructed monolithically with the supporting concrete beams, a portion of the slab acts as the upper flange of the beam. The effective flange width should not exceed (1) one-fourth the span of the beam, (2) the width of the web portion of the beam plus 16 times the thickness of the slab, or (3) the center-to-center distance between beams. T beams where the upper flange is not a portion of a slab should have a flange thickness not less than one-half the width of the web and a flange width not more than four times the width of the web. For preliminary designs, the preceding formulas given for rectangular beams with tensile reinforcing only can be used, because the neutral axis is usually in, or near, the flange. The area of tensile reinforcing is usually critical.

f'_s	п	f_s	K	k	j	р
2000	15	900	175	0.458	0.847	0.0129
2500	12	1125	218	0.458	0.847	0.0161
3000	10	1350	262	0.458	0.847	0.0193
3750	8	1700	331	0.460	0.847	0.0244

TABLE 5.2 Coefficients K, k, j, p for Rectangular Sections[†]

 $^{\dagger}f_s = 16,000 \text{ lb/in}^2 (110 \text{ MPa}).$

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Beams with Tensile and Compressive Reinforcing. Beams with compressive reinforcing are generally used when the size of the beam is limited. The allowable beam dimensions are used in the formulas given earlier to determine the moment that could be carried by a beam without compressive reinforcement. The reinforcing requirements may then be approximately determined from

$$A_s = \frac{8M}{7f_s d} \qquad A_{sc} = \frac{M - M'}{nf_c d}$$

- where $A_s = \text{total cross-sectional area of tensile reinforcing,}$ in² (mm²)
 - $A_{\rm sc} =$ cross-sectional area of compressive reinforcing, in² (mm²)
 - $M = \text{total bending moment, lb} \cdot \text{in (K} \cdot \text{Nm)}$
 - M' = bending moment that would be carried by beam of balanced design and same dimensions with tensile reinforcing only, lb · in (K · Nm)
 - n = ratio of modulus of elasticity of steel to that of concrete

Checking Stresses in Beams. Beams designed using the preceding approximate formulas should be checked to ensure that the actual stresses do not exceed the allowable, and that the reinforcing is not excessive. This can be accomplished by determining the moment of inertia of the beam. In this determination, the concrete below the neutral axis should not be considered as stressed, whereas the reinforcing steel should be transformed into an equivalent concrete section. For tensile reinforcing, this transformation is made

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by multiplying the area A_s by *n*, the ratio of the modulus of elasticity of steel to that of concrete. For compressive reinforcing, the area A_{sc} is multiplied by 2(n - 1). This factor includes allowances for the concrete in compression replaced by the compressive reinforcing and for the plastic flow of concrete. The neutral axis is then located by solving

$$\frac{1}{2}bc_c^2 + 2(n-1)A_{sc}c_{sc} = nA_sc_s$$

for the unknowns c_c , c_{sc} , and c_s (Fig. 5.2). The moment of inertia of the transformed beam section is

$$I = \frac{1}{3}bc_{c}^{3} + 2(n-1)A_{sc}c_{sc}^{2} + nA_{s}c_{s}^{2}$$

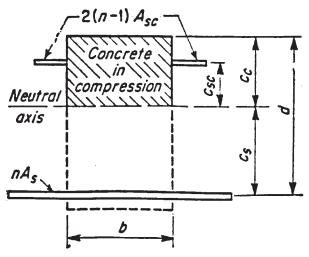


FIGURE 5.2 Transformed section of concrete beam.

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and the stresses are

$$f_c = \frac{Mc_c}{I}$$
 $f_{sc} = \frac{2nMc_{sc}}{I}$ $f_s = \frac{nMc_s}{I}$

where $f_c, f_{sc}, f_s = \text{actual unit stresses in extreme fiber of concrete, in compressive reinforcing steel, and in tensile reinforcing steel, respectively, <math>\text{lb/in}^2$ (MPa)

- $c_c, c_{sc}, c_s =$ distances from neutral axis to face of concrete, to compressive reinforcing steel, and to tensile reinforcing steel, respectively, in (mm)
 - I = moment of inertia of transformed beam section, in⁴ (mm⁴)
 - b = beam width, in (mm)

and A_s , A_{sc} , M, and n are as defined earlier in this chapter.

Shear and Diagonal Tension in Beams. The shearing unit stress, as a measure of diagonal tension, in a reinforced concrete beam is

$$v = \frac{V}{bd}$$

where v = shearing unit stress, lb/in² (MPa)

V =total shear, lb (N)

b = width of beam (for T beam use width of stem), in (mm)

$$d =$$
 effective depth of beam

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If the value of the shearing stress as computed earlier exceeds the allowable shearing unit stress as specified by the ACI *Code*, web reinforcement should be provided. Such reinforcement usually consists of stirrups. The crosssectional area required for a stirrup placed perpendicular to the longitudinal reinforcement is

$$A_v = \frac{(V - V')s}{f_i d}$$

- where $A_v = \text{cross-sectional}$ area of web reinforcement in distance *s* (measured parallel to longitudinal reinforcement), in² (mm²)
 - f_{ν} = allowable unit stress in web reinforcement, lb/in² (MPa)

$$V =$$
total shear, lb (N)

- V' = shear that concrete alone could carry (= $v_c bd$), lb (N)
 - s = spacing of stirrups in direction parallel to that of longitudinal reinforcing, in (mm)
 - d = effective depth, in (mm)

Stirrups should be so spaced that every 45° line extending from the middepth of the beam to the longitudinal tension bars is crossed by at least one stirrup. If the total shearing unit stress is in excess of $3 \sqrt{f'_c}$ lb/in² (MPa), every such line should be crossed by at least two stirrups. The shear stress at any section should not exceed $5 \sqrt{f'_c}$ lb/in² (MPa).

Bond and Anchorage for Reinforcing Bars. In beams in which the tensile reinforcing is parallel to the compression face, the bond stress on the bars is

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	Horizontal bars with more	
	cast below the bar [‡]	Other bars [‡]
Tension bars with sizes and	$\frac{3.4 \sqrt{f_c^2}}{5.6}$ or 350, whichever is less	$\frac{4.8 \sqrt{f_c'}}{50}$ or 500, whichever is less
deformations conforming to	D	D
ASTM A305		
Tension bars with sizes and	$2.1\sqrt{f_c'}$	$3\sqrt{f_c'}$
deformations conforming to		
ASTM A408	I	
Deformed compression bars	$6.5\sqrt{f_c'}$ or 400, whichever is less	$6.5\sqrt{f_c'}$ or 400, whichever is less
	Ĺ	Ĺ
Plain bars	1.7 $\sqrt{f_c'}$ or 160, whichever is less	$2.4\sqrt{f_c'}$ or 160, whichever is less
† lb/in ² (× 0.006895 = MPa).	1 ² (× 0.006895 = MPa).	

compressive strength of concrete, lb/in^2 (MPa); D = nominal diameter of bar, in (mm). II £' ----

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$$u = \frac{V}{jd\Sigma_0}$$

where $u = \text{bond stress on surface of bar, lb/in}^2$ (MPa)

V = total shear, lb (N)

- d = effective depth of beam, in (mm)
- $\Sigma_0 =$ sum of perimeters of tensile reinforcing bars, in (mm)

For preliminary design, the ratio j may be assumed to be 7/8. Bond stresses may not exceed the values shown in Table 5.3.

Columns

The principal columns in a structure should have a minimum diameter of 10 in (255 mm) or, for rectangular columns, a minimum thickness of 8 in (203 mm) and a minimum gross cross-sectional area of 96 in² (61,935 mm²).

Short columns with closely spaced spiral reinforcing enclosing a circular concrete core reinforced with vertical bars have a maximum allowable load of

$$P = A_g(0.25f_c' + f_sp_g)$$

where P = total allowable axial load, lb (N)

 A_g = gross cross-sectional area of column, in² (mm²)

 f'_c = compressive strength of concrete, lb/in² (MPa)

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- f_s = allowable stress in vertical concrete reinforcing, lb/in² (MPa), equal to 40 percent of the minimum yield strength, but not to exceed 30,000 lb/in² (207 MPa)
- p_g = ratio of cross-sectional area of vertical reinforcing steel to gross area of column A_g

The ratio p_g should not be less than 0.01 or more than 0.08. The minimum number of bars to be used is six, and the minimum size is No. 5. The spiral reinforcing to be used in a spirally reinforced column is

$$p_s = 0.45 \left(\frac{A_g}{A_c} - 1\right) \frac{f_c'}{f_y}$$

- where p_s = ratio of spiral volume to concrete-core volume (out-to-out spiral)
 - A_c = cross-sectional area of column core (out-toout spiral), in² (mm²)
 - f_y = yield strength of spiral reinforcement, lb/in² (MPa), but not to exceed 60,000 lb/in² (413 MPa)

The center-to-center spacing of the spirals should not exceed one-sixth of the core diameter. The clear spacing between spirals should not exceed one-sixth the core diameter, or 3 in (76 mm), and should not be less than 1.375 in (35 mm), or 1.5 times the maximum size of coarse aggregate used.

Short Columns with Ties. The maximum allowable load on short columns reinforced with longitudinal bars and separate lateral ties is 85 percent of that given earlier for spirally

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reinforced columns. The ratio p_g for a tied column should not be less than 0.01 or more than 0.08. Longitudinal reinforcing should consist of at least four bars; minimum size is No. 5.

Long Columns. Allowable column loads where compression governs design must be adjusted for column length as follows:

1. If the ends of the column are fixed so that a point of contraflexure occurs between the ends, the applied axial load and moments should be divided by *R* from (*R* cannot exceed 1.0)

$$R = 1.32 - \frac{0.006h}{r}$$

2. If the relative lateral displacement of the ends of the columns is prevented and the member is bent in a single curvature, applied axial loads and moments should be divided by *R* from (*R* cannot exceed 1.0)

$$R = 1.07 - \frac{0.008h}{r}$$

where h = unsupported length of column, in (mm)

r = radius of gyration of gross concrete area, in (mm)

= 0.30 times depth for rectangular column

= 0.25 times diameter for circular column

R =long-column load reduction factor

Applied axial load and moment when tension governs design should be similarly adjusted, except that R varies

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linearly with the axial load from the values given at the balanced condition.

Combined Bending and Compression. The strength of a symmetrical column is controlled by compression if the equivalent axial load N has an eccentricity e in each principal direction no greater than given by the two following equations and by tension if e exceeds these values in either principal direction.

For spiral columns,

$$e_b = 0.43 p_e m D_s + 0.14t$$

For tied columns,

$$e_b = (0.67p_g m + 0.17)d$$

where e = eccentricity, in (mm)

- $e_b = \text{maximum permissible eccentricity, in (mm)}$
- N = eccentric load normal to cross section of column
- p_g = ratio of area of vertical reinforcement to gross concrete area

$$m = f_v / 0.85 f_c'$$

- D_s = diameter of circle through centers of longitudinal reinforcement, in (mm)
 - t = diameter of column or overall depth of column, in (mm)
 - d = distance from extreme compression fiber to centroid of tension reinforcement, in (mm)

$$f_y$$
 = yield point of reinforcement, lb/in² (MPa)

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Design of columns controlled by compression is based on the following equation, except that the allowable load Nmay not exceed the allowable load P, given earlier, permitted when the column supports axial load only:

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_b} + \frac{f_{by}}{F_b} \le 1.0$$

where $f_a = axial$ load divided by gross concrete area, lb/in² (MPa)

- f_{bx}, f_{by} = bending moment about *x* and *y* axes, divided by section modulus of corresponding transformed uncracked section, lb/in² (MPa)
 - F_b = allowable bending stress permitted for bending alone, lb/in² (MPa)

$$F_a = 0.34(1 + p_e m)f'_c$$

The allowable bending load on columns controlled by tension varies linearly with the axial load from M_0 when the section is in pure bending to M_b when the axial load is N_b .

For spiral columns,

$$M_0 = 0.12A_{\rm st}f_{\rm v}D_{\rm s}$$

For tied columns,

$$M_0 = 0.40 A_s f_v (d - d')$$

where $A_{st} = \text{total}$ area of longitudinal reinforcement, in² (mm²)

 f_v = yield strength of reinforcement, lb/in² (MPa)

 D_s = diameter of circle through centers of longitudinal reinforcement, in (mm)

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- A_s = area of tension reinforcement, in² (mm²)
 - d = distance from extreme compression fiber to centroid of tension reinforcement, in (mm)

 N_b and M_b are the axial load and moment at the balanced condition (i.e., when the eccentricity *e* equals e_b as determined). At this condition, N_b and M_b should be determined from

$$M_b = N_b e_b$$

When bending is about two axes,

$$\frac{M_x}{M_{0x}} + \frac{M_y}{M_{0y}} \le 1$$

where M_z and M_y are bending moments about the x and y axes, and M_{0x} and M_{0y} are the values of M_0 for bending about these axes.

PROPERTIES IN THE HARDENED STATE

Strength is a property of concrete that nearly always is of concern. Usually, it is determined by the ultimate strength of a specimen in compression, but sometimes flexural or tensile capacity is the criterion. Because concrete usually gains strength over a long period of time, the compressive strength at 28 days is commonly used as a measure of this property.

The 28-day compressive strength of concrete can be estimated from the 7-day strength by a formula proposed by W. A. Slater:

$$S_{28} = S_7 + 30\sqrt{S_7}$$

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where $S_{28} = 28$ -day compressive strength, lb/in² (MPa), and $S_7 = 7$ -day strength, lb/in² (MPa).

Concrete may increase significantly in strength after 28 days, particularly when cement is mixed with fly ash. Therefore, specification of strengths at 56 or 90 days is appropriate in design.

Concrete strength is influenced chiefly by the water/cement ratio; the higher this ratio is, the lower the strength. The relationship is approximately linear when expressed in terms of the variable C/W, the ratio of cement to water by weight. For a workable mix, without the use of water reducing admixtures,

$$S_{28} = 2700 \frac{C}{W} - 760$$

Tensile strength of concrete is much lower than compressive strength and, regardless of the types of test, usually has poor correlation with f'_c . As determined in flexural tests, the tensile strength (modulus of rupture—not the true strength) is about $7\sqrt{f'_c}$ for the higher strength concretes and $10\sqrt{f'_c}$ for the lower strength concretes.

Modulus of elasticity E_c , generally used in design for concrete, is a secant modulus. In ACI 318, "Building Code Requirements for Reinforced Concrete," it is determined by

$$E_c = w^{1.5} 33 \sqrt{f_c'}$$

where w = weight of concrete, lb/ft³ (kg/m³); and f'_c = specified compressive strength at 28 days, lb/in² (MPa). For normal-weight concrete, with w = 145 lb/ft³ (kg/m³),

$$E_c = 57,000\sqrt{f_c'}$$

The modulus increases with age, as does the strength.

TENSION DEVELOPMENT LENGTHS

For bars and deformed wire in tension, basic development length is defined by the equations that follow. For No. 11 and smaller bars,

$$l_d = \frac{0.04A_b f_y}{\sqrt{f_c'}}$$

where $A_b = \text{area of bar, in}^2 (\text{mm}^2)$

 f_v = yield strength of bar steel, lb/in² (MPa)

 $f_c' = 28$ -day compressive strength of concrete, lb/in^2 (MPa)

However, l_d should not be less than 12 in (304.8 mm), except in computation of lap splices or web anchorage.

For No. 14 bars,

$$l_d = 0.085 \frac{f_y}{\sqrt{f_c'}}$$

For No. 18 bars,

$$l_d = 0.125 \frac{f_y}{\sqrt{f_c'}}$$

and for deformed wire,

$$l_{d} = 0.03d_{b}\frac{f_{y} - 20,000}{\sqrt{f_{c}'}} \ge 0.02\frac{A_{w}}{S_{w}}\frac{f_{y}}{\sqrt{f_{c}'}}$$

where A_w is the area, in² (mm²); and s_w is the spacing, in (mm), of the wire to be developed. Except in computation of

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lap splices or development of web reinforcement, l_d should not be less than 12 in (304.8 mm).

COMPRESSION DEVELOPMENT LENGTHS

For bars in compression, the basic development length l_d is defined as

$$l_{d} = \frac{0.02 f_{y} d_{b}}{\sqrt{f_{c}'}} \ge 0.0003 d_{b} f_{y}$$

but l_d not be less than 8 in (20.3 cm) or $0.0003f_v d_b$.

CRACK CONTROL OF FLEXURAL MEMBERS

Because of the risk of large cracks opening up when reinforcement is subjected to high stresses, the ACI *Code* recommends that designs be based on a steel yield strength f_y no larger than 80 ksi (551.6 MPa). When design is based on a yield strength f_y greater than 40 ksi (275.8 MPa), the cross sections of maximum positive and negative moment should be proportioned for crack control so that specific limits are satisfied by

$$z = f_s \sqrt[3]{d_c A}$$

where f_s = calculated stress, ksi (MPa), in reinforcement at service loads

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- d_c = thickness of concrete cover, in (mm), measured from extreme tension surface to center of bar closest to that surface
- A = effective tension area of concrete, in² (mm²) per bar. This area should be taken as that surrounding main tension reinforcement, having the same centroid as that reinforcement, multiplied by the ratio of the area of the largest bar used to the total area of tension reinforcement

These limits are $z \le 175$ kip/in (30.6 kN/mm) for interior exposures and $z \le 145$ kip/in (25.3 kN/mm) for exterior exposures. These correspond to limiting crack widths of 0.016 to 0.013 in (0.406 to 0.33 mm), respectively, at the extreme tension edge under service loads. In the equation for z, f_s should be computed by dividing the bending moment by the product of the steel area and the internal moment arm, but f_s may be taken as 60 percent of the steel yield strength without computation.

REQUIRED STRENGTH

For combinations of loads, the ACI *Code* requires that a structure and its members should have the following ultimate strengths (capacities to resist design loads and their related internal moments and forces):

With wind and earthquake loads not applied,

$$U = 1.4D + 1.7L$$

where D = effect of basic load consisting of dead load plus volume change (shrinkage, temperature) and L = effect of live load plus impact.

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When wind loads are applied, the largest of the preceeding equation and the two following equations determine the required strength:

$$U = 0.75(1.4D + 1.7L + 1.7W)$$
$$U = 0.9D + 1.3W$$

where W = effect of wind load.

If the structure can be subjected to earthquake forces E, substitute 1.1E for W in the preceding equation.

Where the effects of differential settlement, creep, shrinkage, or temperature change may be critical to the structure, they should be included with the dead load D, and the strength should be at least equal to

$$U = 0.75(1.4D + 1.7L) \ge 1.4(D + T)$$

where T = cumulative effects of temperature, creep, shrinkage, and differential settlement.

DEFLECTION COMPUTATIONS AND CRITERIA FOR CONCRETE BEAMS

The assumptions of working-stress theory may also be used for computing deflections under service loads; that is, elastictheory deflection formulas may be used for reinforced-concrete beams. In these formulas, the *effective moment of inertia* I_c *is given by*

$$I_e = \left(\frac{M_{\rm cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{\rm cr}}{M_a}\right)^3\right] I_{\rm cr} \le I_g$$

where I_g = moment of inertia of the gross concrete section

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 $M_{\rm cr} = {\rm cracking\ moment}$

 M_a = moment for which deflection is being computed

 $I_{\rm cr}$ = cracked concrete (transformed) section

If y_t is taken as the distance from the centroidal axis of the gross section, neglecting the reinforcement, to the extreme surface in tension, the cracking moment may be computed from

$$M_{\rm cr} = \frac{f_r I_g}{y_t}$$

with the modulus of rupture of the concrete $f_r = 7.5\sqrt{f'_c}$.

The deflections thus calculated are those assumed to occur immediately on application of load. Additional long-time deflections can be estimated by multiplying the immediate deflection by 2 when there is no compression reinforcement or by $2 - 1.2A'_s/A_s \ge 0.6$, where A'_s is the area of compression reinforcement.

ULTIMATE-STRENGTH DESIGN OF RECTANGULAR BEAMS WITH TENSION REINFORCEMENT ONLY

Generally, the area A_s of tension reinforcement in a reinforced-concrete beam is represented by the ratio $\rho = A_s/bd$, where *b* is the beam width and *d* is the distance from extreme compression surface to the centroid of tension reinforcement. At ultimate strength, the steel at a critical section of the beam is at its yield strength f_y if the concrete does not fail in compression first. Total tension in the steel then will be $A_s f_y = \rho f_y bd$. It is opposed, by an equal compressive force:

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$$0.85 f_c' ba = 0.85 f_c' b\beta_1 c$$

where $f'_c = 28$ -day strength of the concrete, ksi (MPa)

- a = depth of the equivalent rectangular stress distribution
- c = distance from the extreme compression surface to the neutral axis
- $\beta_1 = a \text{ constant}$

Equating the compression and tension at the critical section yields

$$c = \frac{pf_y}{0.85\beta_1 f_c'} d$$

The criterion for compression failure is that the maximum strain in the concrete equals 0.003 in/in (0.076 mm/mm). In that case,

$$c = \frac{0.003}{f_s/E_s + 0.003} d$$

where f_s = steel stress, ksi (MPa)

 $E_s =$ modulus of elasticity of steel

= 29,000 ksi (199.9 GPa)

Balanced Reinforcing

Under balanced conditions, the concrete reaches its maximum strain of 0.003 when the steel reaches its yield strength f_y . This determines the steel ratio for balanced conditions:

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$$\rho_b = \frac{0.85\beta_1 f_c'}{f_y} \ \frac{87,000}{87,000 + f_y}$$

Moment Capacity

For such underreinforced beams, the bending-moment capacity of ultimate strength is

$$M_u = 0.90[bd^2 f'_c \omega (1 - 0.59\omega)]$$
$$= 0.90 \left[A_s f_y \left(d - \frac{a}{2} \right) \right]$$

where $\omega = \rho f_y / f'_c$ and $a = A_s f_y / 0.85 f'_c$.

Shear Reinforcement

The ultimate shear capacity V_n of a section of a beam equals the sum of the nominal shear strength of the concrete V_c and the nominal shear strength provided by the reinforcement V_s ; that is, $V_n = V_c + V_s$. The factored shear force V_u on a section should not exceed

$$\phi V_n = \phi (V_c + V_s)$$

where ϕ = capacity reduction factor (0.85 for shear and torsion). Except for brackets and other short cantilevers, the section for maximum shear may be taken at a distance equal to *d* from the face of the support.

The shear Vc carried by the concrete alone should not exceed $2\sqrt{f_c'} b_w d$, where bw is the width of the beam web and d, the depth of the centroid of reinforcement. (As an alternative, the maximum for Vc may be taken as

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$$V_c = \left(1.9\sqrt{f_c'} + 2500\rho_w \frac{V_u d}{M_u}\right) b_w d$$
$$\leq 3.5\sqrt{f_c'} b_w d$$

where $\rho_w = A_s / b_w d$ and V_u and M_u are the shear and bending moment, respectively, at the section considered, but M_u should not be less than $V_u d$.)

When V_u is larger than ϕV_c , the excess shear has to be resisted by web reinforcement.

The area of steel required in vertical stirrups, $in^2 (mm^2)$, per stirrup, with a spacing *s*, in (mm), is

$$A_v = \frac{V_s S}{f_y d}$$

where $f_y =$ yield strength of the shear reinforcement. A_v is the area of the stirrups cut by a horizontal plane. V_s should not exceed $8\sqrt{f'_c}b_wd$ in sections with web reinforcement, and f_y should not exceed 60 ksi (413.7 MPa). Where shear reinforcement is required and is placed perpendicular to the axis of the member, it should not be spaced farther apart than 0.5d, or more than 24 in (609.6 mm) c to c. When V_s exceeds $4\sqrt{f'_c}b_wd$, however, the maximum spacing should be limited to 0.25d.

Alternatively, for practical design, to indicate the stirrup spacing *s* for the design shear V_u , stirrup area A_v , and geometry of the member b_w and d,

$$s = \frac{A_v \phi f_y d}{V_u - 2\phi \sqrt{f_c'} b_w d}$$

The area required when a single bar or a single group of parallel bars are all bent up at the same distance from the support at angle α with the longitudinal axis of the member is

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$$A_v = \frac{V_s}{f_y \sin \alpha}$$

in which V_s should not exceed $3\sqrt{f'_c}b_w d$. A_v is the area cut by a plane normal to the axis of the bars. The area required when a series of such bars are bent up at different distances from the support or when inclined stirrups are used is

$$A_{v} = \frac{V_{s}s}{(\sin \alpha + \cos \alpha)f_{v}d}$$

A minimum area of shear reinforcement is required in all members, except slabs, footings, and joists or where V_{μ} is less than $0.5V_c$.

Development of Tensile Reinforcement

At least one-third of the positive-moment reinforcement in simple beams and one-fourth of the positive-moment reinforcement in continuous beams should extend along the same face of the member into the support, in both cases, at least 6 in (152.4 mm) into the support. At simple supports and at points of inflection, the diameter of the reinforcement should be limited to a diameter such that the development length l_d satisfies

$$l_d = \frac{M_n}{V_u} + l_a$$

where M_n = computed flexural strength with all reinforcing steel at section stressed to f_y

- V_u = applied shear at section
- l_a = additional embedment length beyond inflection point or center of support

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At an inflection point, l_a is limited to a maximum of d, the depth of the centroid of the reinforcement, or 12 times the reinforcement diameter.

Hooks on Bars

The basic development length for a hooked bar with $f_y = 60$ ksi (413.7 MPa) is defined as

$$l_{hb} = \frac{1200d_b}{\sqrt{f_c'}}$$

where d_b is the bar diameter, in (mm), and f'_c is the 28-day compressive strength of the concrete, lb/in^2 (MPa).

WORKING-STRESS DESIGN OF RECTANGULAR BEAMS WITH TENSION REINFORCEMENT ONLY

From the assumption that stress varies across a beam section with the distance from the neutral axis, it follows that

$$\frac{nf_c}{f_s} = \frac{k}{1-k}$$

where $n = \text{modular ratio } E_s / E_c$

 $E_s =$ modulus of elasticity of steel reinforcement, ksi (MPa)

 E_c = modulus of elasticity of concrete, ksi (MPa)

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- f_c = compressive stress in extreme surface of concrete, ksi (MPa)
- $f_s =$ stress in steel, ksi (MPa)
- kd = distance from extreme compression surface to neutral axis, in (mm)
 - d = distance from extreme compression to centroid of reinforcement, in (mm)

When the steel ratio $\rho = A_s/bd$, where A_s = area of tension reinforcement, in² (mm²), and *b* = beam width, in (mm), is known, *k* can be computed from

$$k = \sqrt{2n\rho + (n\rho)^2 - n\rho}$$

Wherever positive-moment steel is required, ρ should be at least 200/ f_y , where f_y is the steel yield stress. The distance *jd* between the centroid of compression and the centroid of tension, in (mm), can be obtained from

$$j = 1 - \frac{k}{3}$$

Allowable Bending Moment

The moment resistance of the concrete, in \cdot kip (k \cdot Nm) is

$$M_c = \frac{1}{2} f_c k j b d^2 = K_c b d^2$$

where $K_c = \frac{1}{2} f_c kj$. The moment resistance of the steel is

$$M_s = f_s A_s jd = f_s \rho j bd^2 = K_s bd^2$$

where $K_s = f_s \rho j$.

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Allowable Shear

The nominal unit shear stress acting on a section with shear V is

$$v = \frac{V}{bd}$$

Allowable shear stresses are 55 percent of those for ultimate-strength design. Otherwise, designs for shear by the working-stress and ultimate-strength methods are the same. Except for brackets and other short cantilevers, the section for maximum shear may be taken at a distance d from the face of the support. In working-stress design, the shear stress v_c carried by the concrete alone should not exceed $1.1 \sqrt{f'_c}$. (As an alternative, the maximum for v_c may be taken as $\sqrt{f'_c} + 1300\rho V d/M$, with a maximum of $1.9 \sqrt{f'_c}$; f'_c is the 28-day compressive strength of the concrete, $1b/in^2$ (MPa), and M is the bending moment at the section but should not be less than Vd.)

At cross sections where the torsional stress v_t exceeds $0.825\sqrt{f_c}$, v_c should not exceed

$$v_c = \frac{1.1\sqrt{f_c'}}{\sqrt{1 + (v_t/1.2v)^2}}$$

The excess shear $v - v_c$ should not exceed $4.4\sqrt{f_c'}$ in sections with web reinforcement. Stirrups and bent bars should be capable of resisting the excess shear $V' = V - v_c bd$.

The area required in the legs of a vertical stirrup, in^2 (mm²), is

$$A_v = \frac{V'_s}{f_v d}$$

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where s = spacing of stirrups, in (mm); and $f_v =$ allowable stress in stirrup steel, (lb/in²) (MPa).

For a single bent bar or a single group of parallel bars all bent at an angle α with the longitudinal axis at the same distance from the support, the required area is

$$A_v = \frac{V'}{f_v \sin \alpha}$$

For inclined stirrups and groups of bars bent up at different distances from the support, the required area is

$$A_{v} = \frac{V_{s}'}{f_{v} d(\sin \alpha + \cos \alpha)}$$

Stirrups in excess of those normally required are provided each way from the cutoff for a distance equal to 75 percent of the effective depth of the member. Area and spacing of the excess stirrups should be such that

$$A_v \ge 60 \, \frac{b_w s}{f_y}$$

where $A_v = \text{stirrup cross-sectional area, in}^2 (\text{mm}^2)$

 b_w = web width, in (mm)

s = stirrup spacing, in (mm)

 f_y = yield strength of stirrup steel, (lb/in²) (MPa)

Stirrup spacing *s* should not exceed $d/8\beta_b$, where β_b is the ratio of the area of bars cut off to the total area of tension bars at the section and *d* is the effective depth of the member.

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ULTIMATE-STRENGTH DESIGN OF RECTANGULAR BEAMS WITH COMPRESSION BARS

The bending-moment capacity of a rectangular beam with both tension and compression steel is

$$M_{u} = 0.90 \left[(A_{s} - A'_{s}) f_{y} \left(d - \frac{a}{2} \right) + A'_{s} f_{y} (d - d') \right]$$

where a = depth of equivalent rectangular compressive stress distribution

 $= (A_s - A'_s)f_y/f'_cb$

- b = width of beam, in (mm)
- d = distance from extreme compression surface to centroid of tensile steel, in (mm)
- d' = distance from extreme compression surface to centroid of compressive steel, in (mm)
- A_s = area of tensile steel, in² (mm²)
- A'_s = area of compressive steel, in² (mm²)
- f_v = yield strength of steel, ksi (MPa)
- $f_c' = 28$ -day strength of concrete, ksi (MPa)

This is valid only when the compressive steel reaches f_y and occurs when

$$(\rho - \rho') \ge 0.85 \beta_1 \frac{f'_c d'}{f_y d} \frac{87,000}{87,000 - f_y}$$

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where $\rho = A_s/bd$

$$\rho' = A'_s/bd$$

 $\beta_1 = a \text{ constant}$

WORKING-STRESS DESIGN OF RECTANGULAR BEAMS WITH COMPRESSION BARS

The following formulas, based on the linear variation of stress and strain with distance from the neutral axis, may be used in design:

$$k = \frac{1}{1 + f_s / n f_c}$$

where $f_s =$ stress in tensile steel, ksi (MPa)

- $f_c =$ stress in extreme compression surface, ksi (MPa)
- n =modular ratio, E_s/E_c

$$f_s' = \frac{kd - d'}{d - kd} 2f_s$$

where f'_s = stress in compressive steel, ksi (MPa)

- d = distance from extreme compression surface to centroid of tensile steel, in (mm)
- d' = distance from extreme compression surface to centroid of compressive steel, in (mm)

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The factor 2 is incorporated into the preceding equation in accordance with ACI 318, "Building Code Requirements for Reinforced Concrete," to account for the effects of creep and nonlinearity of the stress–strain diagram for concrete. However, f'_s should not exceed the allowable tensile stress for the steel.

Because total compressive force equals total tensile force on a section,

$$C = C_c + C'_s = T$$

- where C = total compression on beam cross section, kip (N)
 - C_c = total compression on concrete, kip (N) at section

 C'_{s} = force acting on compressive steel, kip (N)

T = force acting on tensile steel, kip (N)

$$\frac{f_s}{f_c} = \frac{k}{2[\rho - \rho'(kd - d')/(d - kd)]}$$

where $\rho = A_s/bd$ and $\rho' = A'_s/bd$.

For reviewing a design, the following formulas may be used:

$$k = \sqrt{2n\left(\rho + \rho'\frac{d'}{d}\right) + n^2(\rho + \rho')^2 - n(\rho + \rho')}$$
$$\bar{z} = \frac{(k^3d/3) + 4n\rho'd'[k - (d'/d)]}{k^2 + 4n\rho'[k - (d'/d)]} \qquad jd = d - \bar{z}$$

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where jd is the distance between the centroid of compression and the centroid of the tensile steel. The moment resistance of the tensile steel is

$$M_s = Tjd = A_s f_s jd$$
 $f_s = \frac{M}{A_s jd}$

where M is the bending moment at the section of beam under consideration. The moment resistance in compression is

$$M_{c} = \frac{1}{2} f_{c} jbd^{2} \left[k + 2n\rho' \left(1 - \frac{d'}{kd} \right) \right]$$
$$f_{c} = \frac{2M}{jbd^{2} \{ k + 2n\rho' [1 - d'/kd)] \}}$$

Computer software is available for the preceding calculations. Many designers, however, prefer the following approximate formulas:

$$M_1 = \frac{1}{2} f_c bkd \left(d - \frac{kd}{3} \right)$$
$$M'_s = M - M_1 = 2 f'_s A'_s (d - d')$$

where M = bending moment

 M'_{s} = moment-resisting capacity of compressive steel

$$M_1$$
 = moment-resisting capacity of concrete

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ULTIMATE-STRENGTH DESIGN OF I AND T BEAMS

When the neutral axis lies in the flange, the member may be designed as a rectangular beam, with effective width b and depth d. For that condition, the flange thickness t will be greater than the distance c from the extreme compression surface to the neutral axis,

$$c = \frac{1.18\omega d}{\beta_1}$$

where $\beta_1 = \text{constant}$

 $\omega = A_s f_v / b df_c'$

 A_s = area of tensile steel, in² (mm²)

 f_v = yield strength of steel, ksi (MPa)

 $f_c' = 28$ -day strength of concrete, ksi (MPa)

When the neutral axis lies in the web, the ultimate moment should not exceed

$$M_{u} = 0.90 \left[(A_{s} - A_{sf}) f_{y} \left(d - \frac{a}{2} \right) + A_{sf} f_{y} \left(d - \frac{t}{2} \right) \right] (8.51)$$

where A_{sf} = area of tensile steel required to develop compressive strength of overhanging flange, in² (mm²) = 0.85(b - b_w)tf_c'/f_y

- b_w = width of beam web or stem, in (mm)
 - a = depth of equivalent rectangular compressive stress distribution, in (mm)

$$= (A_s - A_{sf})f_y / 0.85 f_c' b_w$$

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The quantity $\rho_w - \rho_f$ should not exceed 0.75 ρ_b , where ρ_b is the steel ratio for balanced conditions $\rho_w = A_s/b_w d$, and $\rho_f = A_{sf}/b_w d$.

WORKING-STRESS DESIGN OF I AND T BEAMS

For T beams, effective width of compression flange is determined by the same rules as for ultimate-strength design. Also, for working-stress design, two cases may occur: the neutral axis may lie in the flange or in the web. (For negative moment, a T beam should be designed as a rectangular beam with width b equal to that of the stem.)

If the neutral axis lies in the flange, a T or I beam may be designed as a rectangular beam with effective width *b*. If the neutral axis lies in the web or stem, an I or T beam may be designed by the following formulas, which ignore the compression in the stem, as is customary:

$$k = \frac{I}{1 + f_s / n f_c}$$

- where kd = distance from extreme compression surface to neutral axis, in (mm)
 - d = distance from extreme compression surface to centroid of tensile steel, in (mm)
 - f_s = stress in tensile steel, ksi (MPa)
 - f_c = stress in concrete at extreme compression surface, ksi (MPa)

$$n = \text{modular ratio} = E_s/E_c$$

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Because the total compressive force C equals the total tension T,

$$C = \frac{1}{2} f_c (2kd - t) \frac{bt}{kd} = T = A_s f_s$$
$$kd = \frac{2ndA_s + bt^2}{2nA_s + 2bt}$$

where A_s = area of tensile steel, in² (mm²); and t = flange thickness, in (mm).

The distance between the centroid of the area in compression and the centroid of the tensile steel is

$$jd = d - \overline{z} \qquad \overline{z} = \frac{t(3kd - 2t)}{3(2kd - t)}$$

The moment resistance of the steel is

$$M_s = Tjd + A_s f_s jd$$

The moment resistance of the concrete is

$$M_c = Cjd = \frac{f_c btjd}{2kd} (2kd - t)$$

In design, M_s and M_c can be approximated by

$$M_{s} = A_{s} f_{s} \left(d - \frac{t}{2} \right)$$
$$M_{c} = \frac{1}{2} f_{c} bt \left(d - \frac{t}{2} \right)$$

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derived by substituting d - t/2 for jd and $f_c/2$ for $f_c(1 - t/2kd)$, the average compressive stress on the section.

ULTIMATE-STRENGTH DESIGN FOR TORSION

When the ultimate torsion T_u is less than the value calculated from the T_u equation that follows, the area A_v of shear reinforcement should be at least

$$A_v = 50 \, \frac{b_w s}{f_y}$$

However, when the ultimate torsion exceeds T_u calculated from the T_u equation that follows, and where web reinforcement is required, either nominally or by calculation, the minimum area of closed stirrups required is

$$A_v + 2A_t = \frac{50b_w s}{f_y}$$

where A_t is the area of one leg of a closed stirrup resisting torsion within a distance *s*.

Torsion effects should be considered whenever the ultimate torsion exceeds

$$T_u = \phi \left(0.5 \sqrt{f_c'} \ \Sigma x^2 y \right)$$

where ϕ = capacity reduction factor = 0.85

 T_u = ultimate design torsional moment

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 $\sum x^2 y =$ sum for component rectangles of section of product of square of shorter side and longer side of each rectangle (where T section applies, overhanging flange width used in design should not exceed three times flange thickness)

The torsion T_c carried by the concrete alone should not exceed

$$T_c = \frac{0.8\sqrt{f_c'} \ \Sigma x^2 y}{\sqrt{1 + (0.4V_u/C_t T_u)^2}}$$

where $C_t = b_w d / \Sigma x^2 y$.

Spacing of closed stirrups for torsion should be computed from

$$s = \frac{A_t \phi f_y \alpha_t x_1 y_1}{(T_u - \phi T_c)}$$

where A_t = area of one leg of closed stirrup

- $\alpha_t = 0.66 + 0.33y_1/x_1$ but not more than 1.50
- f_{y} = yield strength of torsion reinforcement
- $x_1 =$ shorter dimension *c* to *c* of legs of closed stirrup
- $y_1 =$ longer dimension *c* to *c* of legs of closed stirrup

The spacing of closed stirrups, however, should not exceed $(x_1 + y_1)/4$ or 12 in (304.8 mm). Torsion reinforcement should be provided over at least a distance of d + b beyond the point where it is theoretically required, where *b* is the beam width.

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At least one longitudinal bar should be placed in each corner of the stirrups. Size of longitudinal bars should be at least No. 3, and their spacing around the perimeters of the stirrups should not exceed 12 in (304.8 mm). Longitudinal bars larger than No. 3 are required if indicated by the larger of the values of *Al* computed from the following two equations:

$$Al = 2A_t \frac{x_1 + y_1}{s}$$
$$Al = \left[\frac{400xs}{f_y} \left(\frac{T_u}{(T_u + V_u/3C_t)}\right) - 2A_t\right] \left(\frac{x_1 + y_1}{s}\right)$$

In the second of the preceding two equations $50b_w s/f_y$ may be substituted for $2A_t$.

The maximum allowable torsion is $T_u = \phi 5 T_c$.

WORKING-STRESS DESIGN FOR TORSION

Torsion effects should be considered whenever the torsion T due to service loads exceeds

$$T = 0.55(0.5 f_c' \Sigma x^2 y)$$

where $\sum x^2 y = \text{sum}$ for the component rectangles of the section of the product of the square of the shorter side and the longer side of each rectangle. The allowable torsion stress on the concrete is 55 percent of that computed from the

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preceding T_c equation. Spacing of closed stirrups for torsion should be computed from

$$s = \frac{3A_t \alpha_t x_1 y_1 f_v}{(v_t - v_{tc}) \Sigma x^2 y}$$

where A_t = area of one leg of closed stirrup

$$\alpha_t = 0.66 + \frac{0.33y_1}{x_1}$$
, but not more than 1.50

 v_{tc} = allowable torsion stress on concrete

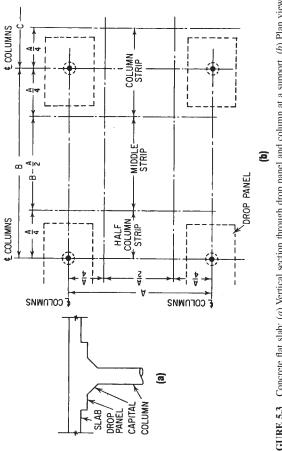
- x_1 = shorter dimension *c* to *c* of legs of closed stirrup
- $y_1 =$ longer dimension *c* to *c* of legs of closed stirrup

FLAT-SLAB CONSTRUCTION

Slabs supported directly on columns, without beams or girders, are classified as flat slabs. Generally, the columns flare out at the top in capitals (Fig. 5.3). However, only the portion of the inverted truncated cone thus formed that lies inside a 90° vertex angle is considered effective in resisting stress. Sometimes, the capital for an exterior column is a bracket on the inner face.

The slab may be solid, hollow, or waffle. A waffle slab usually is the most economical type for long spans, although formwork may be more expensive than for a solid slab. A waffle slab omits much of the concrete that would be in tension and thus is not considered effective in resisting stresses.





Concrete flat slab: (a) Vertical section through drop panel and column at a support. (b) Plan view indicates division of slab into column and middle strips. FIGURE 5.3

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To control deflection, the ACI *Code* establishes minimum thicknesses for slabs, as indicated by the following equation:

$$h = \frac{l_n(0.8 + f_y/200,000)}{36 + 5\beta[\alpha_m - 0.12(1 + 1/\beta)]}$$
$$\geq \frac{l_n(0.8 + f_y/200,000)}{36 + 9\beta}$$

where h = slab thickness, in (mm)

- l_n = length of clear span in long direction, in (mm)
- f_y = yield strength of reinforcement, ksi (MPa)
- β = ratio of clear span in long direction to clear span in the short direction
- α_m = average value of α for all beams on the edges of a panel
 - α = ratio of flexural stiffness $E_{cb}I_b$ of beam section to flexural stiffness $E_{cs}I_s$ of width of slab bounded laterally by centerline of adjacent panel, if any, on each side of beam
- E_{cb} = modulus of elasticity of beam concrete
- E_{cs} = modulus of elasticity of slab concrete
 - I_b = moment of inertia about centroidal axis of gross section of beam, including that portion of slab on each side of beam that extends a distance equal to the projection of the beam above or below the slab, whichever is greater, but not more than four times slab thickness

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 I_s = moment of inertia about centroidal axis of gross section of slab = $h^3/12$ times slab width specified in definition of α

Slab thickness *h*, however, need not be larger than $(l_n/36)$ (0.8 + $f_v/200,000$).

FLAT-PLATE CONSTRUCTION

Flat slabs with constant thickness between supports are called flat plates. Generally, capitals are omitted from the columns.

Exact analysis or design of flat slabs or flat plates is very complex. It is common practice to use approximate methods. The ACI *Code* presents two such methods: direct design and equivalent frame.

In both methods, a flat slab is considered to consist of strips parallel to column lines in two perpendicular directions. In each direction, a *column strip* spans between columns and has a width of one-fourth the shorter of the two perpendicular spans on each side of the column centerline. The portion of a slab between parallel column strips in each panel is called the *middle strip* (see Fig. 5.3).

Direct Design Method

This may be used when all the following conditions exist:

The slab has three or more bays in each direction.

Ratio of length to width of panel is 2 or less.

Loads are uniformly distributed over the panel.

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Ratio of live to dead load is 3 or less.

Columns form an approximately rectangular grid (10 percent maximum offset).

Successive spans in each direction do not differ by more than one-third of the longer span.

When a panel is supported by beams on all sides, the relative stiffness of the beams satisfies

$$0.2 \le \frac{\alpha_1}{\alpha_2} \left(\frac{l_2}{l_1}\right)^2 \le 5$$

where $\alpha_1 = \alpha$ in direction of l_1

- $\alpha_2 = \alpha$ in direction of l_2
- α = relative beam stiffness defined in the preceding equation
- l_1 = span in the direction in which moments are being determined, *c* to *c* of supports
- $l_2 =$ span perpendicular to l_1 , c to c of supports

The basic equation used in direct design is the total static design moment in a strip bounded laterally by the centerline of the panel on each side of the centerline of the supports:

$$M_o = \frac{w l_2 l_n^2}{8}$$

where w = uniform design load per unit of slab area and $l_n =$ clear span in direction moments are being determined.

The strip, with width l_2 , should be designed for bending moments for which the sum in each span of the absolute

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values of the positive and average negative moments equals or exceeds M_o .

1. The sum of the flexural stiffnesses of the columns above and below the slab ΣK_c should be such that

$$\alpha_c = \frac{\Sigma K_c}{\Sigma (K_s + K_b)} \ge \alpha_{\min}$$

where K_c = flexural stiffness of column = $E_{cc}I_c$

- E_{cc} = modulus of elasticity of column concrete
 - I_c = moment of inertia about centroidal axis of gross section of column

$$K_s = E_{cs}I_s$$

$$K_b = E_{cb}I_b$$

- $\alpha_{\min} = \min value \text{ of } \alpha_c \text{ as given in engineering handbooks}$
- **2.** If the columns do not satisfy condition 1, the design positive moments in the panels should be multiplied by the coefficient:

$$\delta_s = 1 + \frac{2 - \beta_a}{4 + \beta_a} \left(1 - \frac{\alpha_c}{\alpha_{\min}} \right)$$

SHEAR IN SLABS

Slabs should also be investigated for shear, both beam type and punching shear. For beam-type shear, the slab is considered

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as a thin, wide rectangular beam. The critical section for diagonal tension should be taken at a distance from the face of the column or capital equal to the effective depth *d* of the slab. The critical section extends across the full width *b* of the slab. Across this section, the nominal shear stress v_u on the unreinforced concrete should not exceed the ultimate capacity $2\sqrt{f'_c}$ or the allowable working stress $1.1\sqrt{f'_c}$, where f'_c is the 28-day compressive strength of the concrete, lb/in² (MPa).

Punching shear may occur along several sections extending completely around the support, for example, around the face of the column or column capital or around the drop panel. These critical sections occur at a distance d/2 from the faces of the supports, where d is the effective depth of the slab or drop panel. Design for punching shear should be based on

$$\phi V_n = \phi (V_c + V_s)$$

where ϕ = capacity reduction factor (0.85 for shear and torsion), with shear strength V_n taken not larger than the concrete strength V_c calculated from

$$V_c = \left(2 + \frac{4}{\beta_c}\right)\sqrt{f_c'} b_o d \le 4\sqrt{f_c'} b_o d$$

where b_o = perimeter of critical section and β_c = ratio of long side to short side of critical section.

However, if shear reinforcement is provided, the allowable shear may be increased a maximum of 50 percent if shear reinforcement consisting of bars is used and increased a maximum of 75 percent if shearheads consisting of two pairs of steel shapes are used.

Shear reinforcement for slabs generally consists of bent bars and is designed in accordance with the provisions for

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beams with the shear strength of the concrete at critical sections taken as $2\sqrt{f'_c} b_o d$ at ultimate strength and $V_n \leq 6\sqrt{f'_c} b_o d$. Extreme care should be taken to ensure that shear reinforcement is accurately placed and properly anchored, especially in thin slabs.

COLUMN MOMENTS

Another important consideration in design of two-way slab systems is the transfer of moments to columns. This is generally a critical condition at edge columns, where the unbalanced slab moment is very high due to the one-sided panel.

The unbalanced slab moment is considered to be transferred to the column partly by flexure across a critical section, which is d/2 from the periphery of the column, and partly by eccentric shear forces acting about the centroid of the critical section.

That portion of unbalanced slab moment M_u transferred by the eccentricity of the shear is given by $\gamma_v M_u$:

$$\gamma_{\nu} = 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{b_1}{b_2}}}$$

where b_1 = width, in (mm), of critical section in the span direction for which moments are being computed; and b_2 = width, in (mm), of critical section in the span direction perpendicular to b_1 .

As the width of the critical section resisting moment increases (rectangular column), that portion of the unbalanced moment transferred by flexure also increases. The

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maximum factored shear, which is determined by combining the vertical load and that portion of shear due to the unbalanced moment being transferred, should not exceed ϕV_c , with V_c given by preceding the V_c equation. The shear due to moment transfer can be determined at the critical section by treating this section as an analogous tube with thickness *d* subjected to a bending moment $\gamma_v M_u$.

The shear stress at the crack, at the face of the column or bracket support, is limited to $0.2f'_c$ or a maximum of 800 A_c , where A_c is the area of the concrete section resisting shear transfer.

The area of shear-friction reinforcement A_{vf} required in addition to reinforcement provided to take the direct tension due to temperature changes or shrinkage should be computed from

$$A_{vf} = \frac{V_u}{\Phi f_v \mu}$$

where V_u is the design shear, kip (kN), at the section; f_y is the reinforcement yield strength, but not more than 60 ksi (413.7 MPa); and μ , the coefficient of friction, is 1.4 for monolithic concrete, 1.0 for concrete placed against hardened concrete, and 0.7 for concrete placed against structural rolled-steel members. The shear-friction reinforcement should be well distributed across the face of the crack and properly anchored at each side.

SPIRALS

This type of transverse reinforcement should be at least $\frac{3}{8}$ in (9.5 mm) in diameter. A spiral may be anchored at each of its ends by $\frac{1}{2}$ extra turns of the spiral. Splices may

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be made by welding or by a lap of 48 bar diameters, but at least 12 in (304.8 mm). Spacing (pitch) of spirals should not exceed 3 in (76.2 mm), or be less than 1 in (25.4 mm). Clear spacing should be at least $1\frac{1}{3}$ times the maximum size of coarse aggregate.

The ratio of the volume of spiral steel/volume of concrete core (out to out of spiral) should be at least

$$\rho_s = 0.45 \left(\frac{A_g}{A_c} - 1 \right) \frac{f_c'}{f_y}$$

where $A_{g} = \text{gross}$ area of column

- $A_c =$ core area of column measured to outside of spiral
- $f_{\rm v}$ = spiral steel yield strength
- $f_c' = 28$ -day compressive strength of concrete

BRACED AND UNBRACED FRAMES

As a guide in judging whether a frame is braced or unbraced, note that the commentary on ACI 318–83 indicates that a frame may be considered braced if the bracing elements, such as shear walls, shear trusses, or other means resisting lateral movement in a story, have a total stiffness at least six times the sum of the stiffnesses of all the columns resisting lateral movement in that story.

The slenderness effect may be neglected under the two following conditions:

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For columns braced against sidesway, when

$$\frac{kl_u}{r} < 34 - 12 \frac{M_1}{M_2}$$

where M_1 = smaller of two end moments on column as determined by conventional elastic frame analysis, with positive sign if column is bent in single curvature and negative sign if column is bent in double curvature; and M_2 = absolute value of larger of the two end moments on column as determined by conventional elastic frame analysis.

For columns not braced against sidesway, when

$$\frac{kl_u}{r} < 22$$

LOAD-BEARING WALLS

These are subject to axial compression loads in addition to their own weight and, where there is eccentricity of load or lateral loads, to flexure. Load-bearing walls may be designed in a manner similar to that for columns but including the design requirements for non-load-bearing walls.

As an alternative, load-bearing walls may be designed by an empirical procedure given in the ACI *Code* when the eccentricity of the resulting compressive load is equal to or less than one-sixth the thickness of the wall.

Load-bearing walls designed by either method should meet the minimum reinforcing requirements for non-loadbearing walls.

In the empirical method the axial capacity, kip (kN), of the wall is

$$\Phi P_n = 0.55 \Phi f_c' A_g \left[1 - \left(\frac{kl_c}{32h}\right)^2 \right]$$

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where
$$f'_c = 28$$
-day compressive strength of concrete,
ksi (MPa)

$$A_g$$
 = gross area of wall section, in² (mm²)

 ϕ = strength reduction factor = 0.70

 l_c = vertical distance between supports, in (mm)

h = overall thickness of wall, in (mm)

k = effective-length factor

For a wall supporting a concentrated load, the length of wall effective for the support of that concentrated load should be taken as the smaller of the distance center to center between loads and the bearing width plus 4h.

SHEAR WALLS

Walls subject to horizontal shear forces in the plane of the wall should, in addition to satisfying flexural requirements, be capable of resisting the shear. The nominal shear stress can be computed from

$$v_u = \frac{V_u}{\phi hd}$$

where V_u = total design shear force

 ϕ = capacity reduction factor = 0.85

 $d = 0.8 l_w$

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- h = overall thickness of wall
- l_w = horizontal length of wall

The shear V_c carried by the concrete depends on whether N_u , the design axial load, lb (N), normal to the wall horizontal cross section and occurring simultaneously with V_u at the section, is a compression or tension force. When N_u is a compression force, V_c may be taken as $2\sqrt{f_c} hd$, where f'_c is the 28-day strength of concrete, lb/in² (MPa). When N_u is a tension force, V_c should be taken as the smaller of the values calculated from

$$V_{c} = 3.3 \sqrt{f_{c}'} h d - \frac{N_{u} d}{4l_{w}}$$
$$V_{c} = h d \left[0.6 \sqrt{f_{c}'} + \frac{l_{w} (1.25 \sqrt{f_{c}'} - 0.2N_{u}/l_{w} h)}{M_{u}/V_{u} - l_{w}/2} \right]$$

This equation does not apply, however, when $M_u/V_u - l_w/2$ is negative.

When the factored shear V_u is less than $0.5\phi V_c$, reinforcement should be provided as required by the empirical method for bearing walls.

When V_u exceeds $0.5\varphi V_c$, horizontal reinforcement should be provided with $V_s = A_v f_y d/s_2$, where s_2 = spacing of horizontal reinforcement, and A_v = reinforcement area. Also, the ratio ρ_h of horizontal shear reinforcement, to the gross concrete area of the vertical section of the wall should be at least 0.0025. Spacing of horizontal shear bars should not exceed $l_w/5$, 3h, or 18 in (457.2 mm). In addition, the ratio of vertical shear reinforcement area to gross concrete area of the horizontal section of wall does not need to be greater than that required for horizontal reinforcement but should not be less than

$$\rho_n = 0.0025 + 0.5 \left(2.5 - \frac{h_w}{l_w} \right)$$

$$(\rho_h - 0.0025) \le 0.0025$$

where $h_w =$ total height of wall. Spacing of vertical shear reinforcement should not exceed $l_w/3$, 3h, or 18 in (457.2 mm).

In no case should the shear strength V_n be taken greater than $10\sqrt{f'_c} hd$ at any section.

Bearing stress on the concrete from anchorages of posttensioned members with adequate reinforcement in the end region should not exceed f_b calculated from

$$f_b = 0.8 f_c' \sqrt{\frac{A_b}{A_b} - 0.2} \le 1.25 f_{ci}'$$
$$f_b = 0.6 \sqrt{f_c'} \sqrt{\frac{A_b}{A_b'}} \le f_c'$$

where A_b = bearing area of anchor plate, and A'_b = maximum area of portion of anchorage surface geometrically similar to and concentric with area of anchor plate.

A more refined analysis may be applied in the design of the end-anchorage regions of prestressed members to develop the ultimate strength of the tendons. ϕ should be taken as 0.90 for the concrete.

CONCRETE GRAVITY RETAINING WALLS

Forces acting on gravity walls include the weight of the wall, weight of the earth on the sloping back and heel, lateral earth pressure, and resultant soil pressure on the base. It is advisable to include a force at the top of the wall to account for

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frost action, perhaps 700 lb/linear ft (1042 kg/m). A wall, consequently, may fail by overturning or sliding, overstressing of the concrete or settlement due to crushing of the soil.

Design usually starts with selection of a trial shape and dimensions, and this configuration is checked for stability. For convenience, when the wall is of constant height, a 1-ft (0.305 m) long section may be analyzed. Moments are taken about the toe. The sum of the righting moments should be at least 1.5 times the sum of the overturning moments. To prevent sliding,

$$\mu R_{\nu} \geq 1.5 P_h$$

where μ = coefficient of sliding friction

 R_v = total downward force on soil, lb (N)

 P_h = horizontal component of earth thrust, lb (N)

Next, the location of the vertical resultant R_v should be found at various sections of the wall by taking moments about the toe and dividing the sum by R_v . The resultant should act within the middle third of each section if there is to be no tension in the wall.

Finally, the pressure exerted by the base on the soil should be computed to ensure that the allowable pressure is not exceeded. When the resultant is within the middle third, the pressures, lb/ft² (Pa), under the ends of the base are given by

$$p = \frac{R_{\nu}}{A} \pm \frac{Mc}{I} = \frac{R_{\nu}}{A} \left(1 \pm \frac{6e}{L}\right)$$

where $A = \text{area of base, ft}^2 (\text{m}^2)$

L = width of base, ft (m)

e = distance, parallel to *L*, from centroid of base to R_v , ft (m)

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Figure 5.4 shows the pressure distribution under a 1-ft (0.305-m) strip of wall for e = L/2 - a, where *a* is the distance of R_v from the toe. When R_v is exactly L/3 from the toe, the pressure at the heel becomes zero (Fig. 5.4*c*). When

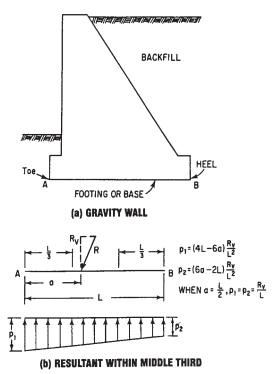


FIGURE 5.4 Diagrams for pressure of the base of a concrete gravity wall on the soil below. (*a*) Vertical section through the wall. (*b*) Significant compression under the entire base.

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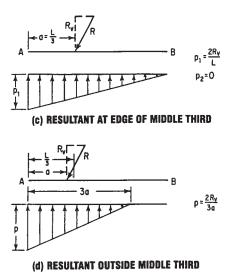
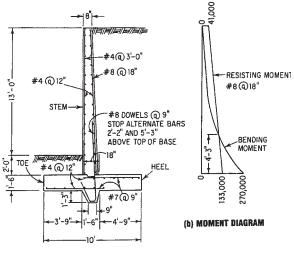


FIGURE 5.4 (*Continued*) Diagrams for pressure of the base of a concrete wall on the soil below. (c) No compression along one edge of the base. (d) Compression only under part of the base. No support from the soil under the rest of the beam.

 R_{ν} falls outside the middle third, the pressure vanishes under a zone around the heel, and pressure at the toe is much larger than for the other cases (Fig. 5.4*d*).

CANTILEVER RETAINING WALLS

This type of wall resists the lateral thrust of earth pressure through cantilever action of a vertical stem and horizontal



(a) TYPICAL WALL SECTION

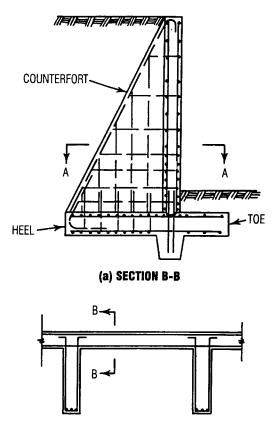
FIGURE 5.5 Cantilever retaining wall. (*a*) Vertical section shows main reinforcing steel placed vertically in the stem. (*b*) Moment diagram.

base (Fig. 5.5). Cantilever walls generally are economical for heights from 10 to 20 ft (3 to 6 m). For lower walls, gravity walls may be less costly; for taller walls, counterforts (Fig. 5.6) may be less expensive.

Shear unit stress on a horizontal section of a counterfort may be computed from $v_c = V_1/bd$, where b is the thickness of the counterfort and d is the horizontal distance from face of wall to main steel,

$$V_1 = V - \frac{M}{d} (\tan \theta + \tan \phi)$$

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(b) PLAN A-A

FIGURE 5.6 Counterfort retaining wall. (*a*) Vertical section. (*b*) Horizontal section.

where V = shear on section

- M = bending moment at section
 - θ = angle earth face of counterfort makes with vertical
- ϕ = angle wall face makes with vertical

For a vertical wall face, $\phi = 0$ and $V_1 = V - (M/d)\tan \theta$. The critical section for shear may be taken conservatively at a distance up from the base equal to $d' \sin \theta \cos \theta$, where d' is the depth of counterfort along the top of the base.

WALL FOOTINGS

The spread footing under a wall (Fig. 5.7) distributes the wall load horizontally to preclude excessive settlement.

The footing acts as a cantilever on opposite sides of the wall under downward wall loads and upward soil pressure. For footings supporting concrete walls, the critical section for bending moment is at the face of the wall; for footings under masonry walls, halfway between the middle and edge of the wall. Hence, for a 1-ft (0.305-m) long strip of symmetrical concrete-wall footing, symmetrically loaded, the maximum moment, ft·lb (N·m), is

$$M = \frac{p}{8} \left(L - a\right)^2$$

where p = uniform pressure on soil, lb/ft² (Pa)

L = width of footing, ft (m)

a = wall thickness, ft (m)

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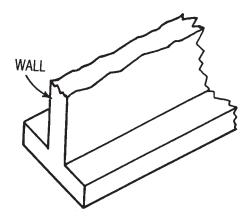


FIGURE 5.7 Concrete wall footing.

If the footing is sufficiently deep that the tensile bending stress at the bottom, $6M/t^2$, where *M* is the factored moment and *t* is the footing depth, in (mm), does not exceed $5\phi\sqrt{f'_c}$, where f'_c is the 28-day concrete strength, $1b/in^2$ (MPa) and $\phi = 0.90$, the footing does not need to be reinforced. If the tensile stress is larger, the footing should be designed as a 12-in (305-mm) wide rectangular, reinforced beam. Bars should be placed across the width of the footing, 3 in (76.2 mm) from the bottom. Bar development length is measured from the point at which the critical section for moment occurs. Wall footings also may be designed by ultimatestrength theory.

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